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We calculate the  $P^0 \rightarrow \gamma\gamma$ ,  $V^0 \rightarrow P^0\gamma$  and  $V^0 \rightarrow V'^0\gamma\gamma$  decays in the framework of a  $U(3) \otimes U(3)$  linear sigma model which includes constituent quarks. For the first two decays this approach improves results based on the anomalous Wess-Zumino term, with contributions due to  $SU(3)$  symmetry breaking and vector mixing. The  $\phi \rightarrow (\omega, \rho)\gamma\gamma$  decays are dominated by resonant  $\eta'$  exchange. Our calculation for the later decays improves and update similar calculations in the -closely related- framework of vector meson dominance. We obtain  $BR(\phi \rightarrow \rho\gamma\gamma) = 2.5 \times 10^{-5}$  and  $BR(\phi \rightarrow \omega\gamma\gamma) = 2.8 \times 10^{-6}$  within the scope of the high-luminosity  $\phi$  factories.

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## I. INTRODUCTION

Radiative decays of mesons have been a useful tool in the past in the underlying of the dynamics of non-perturbative QCD. In spite of being one of the earliest tests for the internal structure of hadrons, the interest on this kind of processes is by no means exhausted. Indeed, there exists renewed interest on vector meson radiative decays [1] due to the possibilities opened by the high luminosity  $e^+e^-$  machines which are allowing us to study rare decay modes of vector mesons. In particular many modes have been recently measured by the experimental groups at Novosibirsk [2] and KLOE Coll. at Frascati  $\phi$  factory started to release even more accurate data [3,4]. Hopefully this data will help us in discriminating the different proposals for the dynamics arising from QCD which governs the different channels.

In the following we study the specific processes  $P^0 \rightarrow \gamma\gamma$ ,  $V^0 \rightarrow P^0\gamma$  and  $V^0 \rightarrow V'^0\gamma\gamma$  decays. In QCD the  $P^0 \rightarrow \gamma\gamma$  and  $V^0 \rightarrow P^0\gamma$  decays are induced by the contributions of external vector currents to the axial anomalies which can be summarized in the Wess-Zumino term [5]. The  $V^0 \rightarrow V'^0\gamma\gamma$  decays were previously calculated [6] in the framework of heavy vector chiral perturbation theory (HVCHPT) [7]. In this theory the  $V^0 \rightarrow V'^0\gamma\gamma$  decays are induced by loops of charged pseudoscalar mesons and the corresponding branching ratios turn out to be rather small ( $\approx 10^{-9}$ ). The importance of intermediate pseudoscalar contributions to the same processes was noticed in [8]. In this picture the  $V^0 \rightarrow V'^0\gamma\gamma$  decays go through the chain  $V^0 \rightarrow P^0\gamma \rightarrow V'^0\gamma\gamma$  where the anomalous decays  $V^0 \rightarrow P^0\gamma$  play a prominent rol. The most important effect here is the possibility of resonant contributions which enhance the corresponding branching ratios and in the case of  $\phi \rightarrow \rho\gamma\gamma$  brings it close to the upper limit encountered by the CMD-2 Collaboration [9]. Indeed, the branching ratio calculated in [8] for this decay is  $\mathcal{B} = 1.3 \times 10^{-4}$  to be compared with the upper bound reported in [9]  $\mathcal{B}_{exp} < 5 \times 10^{-4}$ . The possibility of improving this bound at DAΦNE makes worthy to reconsider these decays.

In this work we study the  $P^0 \rightarrow \gamma\gamma$ ,  $V^0 \rightarrow P^0\gamma$  and  $V^0 \rightarrow V'^0\gamma\gamma$  decays in the framework of a  $U(3) \otimes U(3)$  linear sigma model comprising  $U_A(1)$  symmetry breaking due to strong dynamics and including constituent quarks. This is a generalization to the  $U(3) \otimes U(3)$  version of the  $U(2) \otimes U(2)$  model used in [10] to calculate the  $\omega \rightarrow \rho\pi$  transition and the  $\omega \rightarrow 3\pi$  decay. In this model the contributions to the axial anomalies due to external vector fields can be explicitly calculated yielding sharp predictions for the  $P^0 \rightarrow \gamma\gamma$  and  $V^0 \rightarrow P^0\gamma$  decays since we work with the physical meson fields. The advantage of the explicit introduction of fermion fields is that, on the one hand it makes possible to study effects due to  $SU(3)$  symmetry breaking and on the other hand it also allow us to study the contribution of resonant intermediate pseudoscalar states to more involved decays such as  $V^0 \rightarrow V'^0\gamma\gamma$ .

## II. $U(3) \times U(3)$ LINEAR SIGMA MODEL WITH QUARKS ( $QL\sigma M$ )

The generalization of the model presented in [10] to its  $U(3) \otimes U(3)$  version can be done by studying the  $U_L(3) \otimes U_R(3)$  invariants constructed from  $B = S + iP$ . These invariants were studied long ago [11]

$$X = \text{tr}(BB^\dagger), \quad Y = \text{tr}(BB^\dagger)^2, \quad Z = \det B + \det B^\dagger. \quad (1)$$

The first two terms are  $U_L(3) \otimes U_R(3)$  invariant whereas the  $Z$  term explicitly breaks the  $U_A(1)$  symmetry but respects the  $SU_A(3) \times U_V(3)$  symmetry. The generalization of the Lagrangian presented in [10] is

$$L = \bar{q}[iD_\mu \gamma^\mu + \sqrt{2}g(S + i\gamma_5 P)]q + \frac{1}{2}\text{tr}(D_\mu B D^\mu B^\dagger) - \frac{\mu^2}{2}\text{tr}(B B^\dagger) - \frac{\lambda}{4}\text{tr}(B B^\dagger)^2 - \frac{\lambda'}{4}(\text{tr}(B B^\dagger))^2 + \text{tr}(CS) - \beta(\det B + \det B^\dagger). \quad (2)$$

where the scalar and pseudoescalar fields are given by  $S = \frac{1}{\sqrt{2}}\lambda^i s^i$ ,  $P = \frac{1}{\sqrt{2}}\lambda^i p^i$ , with  $\lambda^i$   $i = 1 \dots 7$  the conventional Gell-Mann matrices and we use the flavor matrices  $\lambda_{\text{ns}} = \text{Diag}(1, 1, 0)$  and  $\lambda_s = \sqrt{2}\text{Diag}(0, 0, 1)$ . Explicitly, the scalar, pseudoscalar and vector fields are

$$S = \begin{pmatrix} \frac{\sigma_{ns} + a^0}{\sqrt{2}} & a^+ & \kappa^+ \\ a^- & \frac{\sigma_{ns} - a^0}{\sqrt{2}} & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & \sigma_s \end{pmatrix} \quad P = \begin{pmatrix} \frac{\eta_{ns} + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_{ns} - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_s \end{pmatrix} \quad V_\mu = \begin{pmatrix} \frac{V_{ns} + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{V_{ns} - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & V_s \end{pmatrix}_\mu. \quad (3)$$

Vector mesons are introduced as external fields through the covariant derivatives

$$D_\mu q \equiv \left( \partial_\mu + i \frac{g_v}{\sqrt{2}} V_\mu \right) q \quad D_\mu B \equiv \partial_\mu B + i \frac{g_v}{\sqrt{2}} [V_\mu, B]. \quad (4)$$

The mesonic sector of this theory has been analyzed in detail in [12–14]. In particular, the vacuum expectation values (vevs) of scalars  $a \equiv \frac{\langle \sigma_{ns} \rangle}{\sqrt{2}}$ ,  $b \equiv \langle \sigma_s \rangle$  are related to the pseudoscalar decay constants as

$$f_\pi = \sqrt{2}a \quad f_k = \frac{1}{\sqrt{2}}(a + b). \quad (5)$$

As for the fermionic sector, quarks acquire mass due to the spontaneous breaking of chiral symmetry. The constituent quark mass matrix generated by this mechanism is  $M_q = \sqrt{2}g \langle S \rangle$ , where  $\langle S \rangle = V = \text{diag}(a, a, b)$ . Explicitly

$$m_u = m_d = \sqrt{2}ga = gf_\pi \quad m_s = \sqrt{2}gb = g(2f_k - f_\pi). \quad (6)$$

In the calculations of the physical processes to be considered below, the free parameters  $g, m_q, m_s$  ( $q = u, d$ ) enter in the combinations  $g/m_q$ ,  $g/m_s$  which are determined by the weak decay constants of pseudoscalars as

$$\frac{g}{m_q} = \frac{1}{f_\pi} \quad \frac{g}{m_s} = \frac{1}{2f_k - f_\pi}. \quad (7)$$

The free parameter  $g_v$  appearing in the covariant derivative (4) can be fixed invoking vector meson dominance of the electromagnetic form factors of *constituent quarks*, i.e. VMD implemented at the constituent quark level (QVMD), or directly from the  $\rho \rightarrow \pi^+ \pi^-$  decay. QVMD hypothesis was shown to work fairly well in the  $U(2) \otimes U(2)$  version of the model [10]. Under this hypothesis, the isovector contribution to the  $qq\gamma$  interaction is mediated by the rho meson. The corresponding amplitude is

$$\mathcal{M}_{VMD} = \epsilon^\alpha f_{\rho\gamma} \left( \frac{-g^{\alpha\nu} + q^\alpha q^\nu / m_\rho^2}{q^2 - m_\rho^2} \right) \frac{g_v}{2} \bar{u} \gamma_\nu u \quad (8)$$

where  $\epsilon$  stands for the photon polarization vector and  $f_{\rho\gamma}$  denotes the  $\rho - \gamma$  coupling. On the other hand, the amplitude for the  $qq\gamma$  transition can be characterized on the base of Lorentz covariance, gauge invariance, parity etc. as

$$\mathcal{M} = \epsilon^\mu \bar{q} \left[ e_q F_1(q^2) \gamma^\mu + \frac{F_2(q^2)}{m_q} q_\nu \sigma^{\mu\nu} \right] q \quad (9)$$

where  $e_q$  stands for the quark charge and  $F_1(q^2)$ ,  $F_2(q^2)$  are form factors which can not be fixed on symmetry arguments alone. Eq. (8) has been written such that  $F_1(q^2)$  is normalized to  $F_1(0) = 1$ . In the soft limit ( $q_\mu \rightarrow 0$ ) only the charge form factor contributes and comparing with Eq.(8) we obtain

$$g_v = \frac{em_\rho^2}{f_{\rho\gamma}}. \quad (10)$$

It still remain to fix  $f_{\rho\gamma}$ . This parameter can be extracted from the measured  $\rho \rightarrow e^+e^-$  decay. Using the central value for the measured branching ratio for this channel [18] we obtain  $g_v = 5.0$ . As a consistency check for this estimate we can evaluate the isoscalar contribution to the  $qq\gamma$  interaction mediated by the  $\omega$  meson. A similar analysis for the isoscalar form factor yields

$$g_v = \frac{em_\omega^2}{3f_{\omega\gamma}}. \quad (11)$$

Extracting the  $\omega - \gamma$  coupling from the  $\omega \rightarrow e^+e^-$  decay we obtain  $g_v = 5.6$ , close to the previous estimate. A calculation for the electromagnetic form factor of the  $s$  quark yields a similar value ( $g_v = 6.0$ ). Finally the coupling  $g_v$  can also be directly extracted from the  $\rho^0 \rightarrow \pi^+\pi^-$  decay. From the Lagrangian (2) we get  $g_{\rho\pi\pi} = g_v$ . Using the measured branching ratio for  $\rho^0 \rightarrow \pi^+\pi^-$  we obtain  $g_v = 6.0$ , fairly consistent with the former estimates. In the whole, the vector coupling lies in the range  $g_v \in [5.0, 6.0]$ . We will use  $g_v = 5.5 \pm 0.5$  in the numerical calculations below.

### III. $P \rightarrow \gamma\gamma$ AND $V \rightarrow P\gamma$ DECAYS

#### A. $P^0(p) \rightarrow \gamma(k_1, \varepsilon_1)\gamma(k_2, \varepsilon_2)$ decays.

The  $P^0 \rightarrow \gamma\gamma$  decays are induced by quark loops in this model. These contributions are finite and we obtain sharp predictions for these decays. The invariant amplitude for the  $\pi^0 \rightarrow \gamma\gamma$  decay as calculated in the model is

$$\mathcal{M}(\pi^0 \rightarrow \gamma\gamma) = C_{\pi\gamma\gamma} \epsilon(\varepsilon_1, \varepsilon_2, k_1, k_2) I(p^2) \quad (12)$$

where  $\epsilon(p, q, r, l)$  is a shorthand notation for  $\epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu r^\rho l^\sigma$  and

$$C_{\pi\gamma\gamma} = 8im_q g N_c (e_u^2 - e_d^2), \quad (13)$$

with  $N_c$  the number of colors. The loop integral is given by

$$I(p^2) = \frac{-i}{16\pi^2} \int_0^1 dx \int_0^{1-x} \frac{dy}{m_q^2 \left[ 1 - x(1-x) \frac{p^2}{m_q^2} + xy \frac{p^2}{m_q^2} - i\varepsilon \right]}. \quad (14)$$

In the chiral limit ( $p^2 = m_\pi^2 = 0$ ) this integral yields

$$I = \frac{-i}{32\pi^2 m_q^2}. \quad (15)$$

and the invariant amplitude in the chiral limit reduces to

$$\mathcal{M}(\pi^0 \rightarrow \gamma\gamma) = \frac{\alpha}{\pi f_\pi} \epsilon(\varepsilon_1, \varepsilon_2, k_1, k_2) \quad (16)$$

where we used  $\frac{g}{m_q} = \frac{1}{f_\pi}$ . This result coincides with the amplitude calculated from the Wess-Zumino term<sup>1</sup>. This amplitude yields the decay width

$$\Gamma(\pi^0 \rightarrow \gamma\gamma)_{QL\sigma M} = \frac{\alpha^2 m_\pi^3}{64\pi^3 f_\pi^2} = 7.63 \text{ eV} \quad (17)$$

to be compared with the experimental result  $\Gamma(\pi^0 \rightarrow \gamma\gamma)_{exp} = (7.79 \pm 0.56) \text{ eV}$ .

As for the  $\eta \rightarrow \gamma\gamma$  and  $\eta' \rightarrow \gamma\gamma$  we must take care of the mixing of the  $\eta_{ns}$  and  $\eta_s$  in this case. The amplitudes for the physical processes are related to those for the flavor fields as follows

$$\begin{aligned} \mathcal{M}_{\eta\gamma\gamma} &= \mathcal{M}_{\eta_{ns}\gamma\gamma} \cos \phi_p - \mathcal{M}_{\eta_s\gamma\gamma} \sin \phi_p \\ \mathcal{M}_{\eta'\gamma\gamma} &= \mathcal{M}_{\eta_{ns}\gamma\gamma} \sin \phi_p + \mathcal{M}_{\eta_s\gamma\gamma} \cos \phi_p. \end{aligned} \quad (18)$$

The calculation of the amplitude for  $\eta_{ns} \rightarrow \gamma\gamma$  goes along the lines of the case of the  $\pi^0$ . We obtain

$$\mathcal{M}_{\eta_{ns}\gamma\gamma} = C_{\eta_{ns}\gamma\gamma} \epsilon(\varepsilon_1, \varepsilon_2, k_1, k_2) I(m_{\eta_{ns}}^2) \quad (19)$$

where

$$C_{\eta_{ns}\gamma\gamma} = 8im_q g N_c (e_u^2 + e_d^2). \quad (20)$$

In the chiral limit  $I$  is given by (15) and the invariant amplitude reads

$$\mathcal{M}(\eta_{ns} \rightarrow \gamma\gamma) = \frac{5\alpha}{3\pi f_\pi} \epsilon(\varepsilon_1, \varepsilon_2, k_1, k_2) \quad (21)$$

where we used Eq. (7). Similar calculations for the  $\eta_s \rightarrow \gamma\gamma$  transition yield

$$\mathcal{M}(\eta_s \rightarrow \gamma\gamma) = \frac{\sqrt{2}\alpha}{3\pi(2f_k - f_\pi)} \epsilon(\varepsilon_1, \varepsilon_2, k_1, k_2) \quad (22)$$

Finally, the invariant amplitudes for the physical processes  $\eta \rightarrow \gamma\gamma$  y  $\eta' \rightarrow \gamma\gamma$  are

$$\begin{aligned} \mathcal{M}_{\eta\gamma\gamma} &= \frac{\alpha}{3\pi f_\pi} \left( 5 \cos \phi_p - \frac{\sqrt{2}f_\pi}{2f_k - f_\pi} \sin \phi_p \right) \epsilon(\varepsilon_1, \varepsilon_2, k_1, k_2) \\ \mathcal{M}_{\eta'\gamma\gamma} &= \frac{\alpha}{3\pi f_\pi} \left( 5 \sin \phi_p + \frac{\sqrt{2}f_\pi}{2f_k - f_\pi} \cos \phi_p \right) \epsilon(\varepsilon_1, \varepsilon_2, k_1, k_2) \end{aligned} \quad (23)$$

Notice that these amplitudes reproduce the results of an effective theory of mesons in which the contributions of the electromagnetic external sources to the axial anomalies are introduced using 't Hooft's anomaly matching conditions [15]. As discussed in [15] from Eqs.(23) we obtain  $\phi_p \in [38.4^\circ, 41.0^\circ]$ , consistent with other estimates [16] and with recent measurements by the KLOE Coll.  $\phi_p^{exp} = 41.8_{-1.6}^{+1.9}$  [3].

## B. $V(q, \eta) \rightarrow P(p)\gamma(k, \varepsilon)$ decays

The most general form of the invariant amplitude for the  $V(q, \eta) \rightarrow P(p)\gamma(k, \varepsilon)$  decays is dictated by Lorentz covariance, gauge invariance, parity etc. as

$$\mathcal{M} = A_{vp} \epsilon(\varepsilon, \eta, k, q) \quad (24)$$

where  $A_{vp}$  is a form factor. The decay widths, written in terms of these form factors are

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<sup>1</sup>This is hardly a surprising result since the anomaly depend on the fermion content of the theory. The anomaly matching condition by 't Hooft used e.g. in [15], is satisfied here due to the same fermionic content in the model and in QCD.

$$\Gamma(V \rightarrow P\gamma) = \frac{|A_{vp}|^2 m_v^3}{96\pi} \left[ 1 - \left( \frac{m_p}{m_v} \right)^2 \right]^3. \quad (25)$$

The value of these form factors on the mass-shell can be extracted from the existing measurements for the corresponding decay widths against which the predictions of particular models must be compared. In the  $QL\sigma M$  the  $V^0 \rightarrow P^0\gamma$  decays are induced by quark loops. The importance of constituent quark loops in the description of  $PVV$  vertices was stressed in [17]. The calculation of the amplitude for the  $\rho^0 \rightarrow \pi^0\gamma$  decay in the  $QL\sigma M$  yields

$$\mathcal{M} = A_{\rho\pi}^{QL\sigma M} \epsilon(\eta, \varepsilon, k, q) \quad (26)$$

where

$$A_{\rho\pi}^{QL\sigma M} = 4m_q g_v g N_c (e_u + e_d) I(m_\rho^2). \quad (27)$$

In this case the loop integral contains spurious imaginary parts coming from unphysical on-shell  $\bar{q}q$  channel in the loops. This is due to the lack of a confining mechanism for the constituent quarks. We approximate this integral to its value at  $q^2 = 0$  and assume that this value is not too different from its value at the physical point in the presence of confinement. This is a reasonable assumption which, as we shall see below, is supported by experimental data. Under this assumption the value predicted by the  $QL\sigma M$  for  $A_{\rho\pi}$  is

$$A_{\rho\pi}^{QL\sigma M} = \frac{eg_v}{8\pi^2 f_\pi} \quad (28)$$

In a similar way we calculate the kinematically allowed  $V \rightarrow P^0\gamma$  and  $P^0 \rightarrow V^0\gamma$  decays, taking care of the mixing between the  $V_{ns}$  and  $V_s$  to form the physical  $\phi$  and  $\omega$ . Our results are shown in Table I where we defined

$$A = \frac{eg_v g}{8\pi^2 m_q} = \frac{eg_v}{8\pi^2 f_\pi}, \quad B = \frac{3eg_v g}{8\pi^2 m_q} = \frac{3eg_v}{8\pi^2 f_\pi}, \quad C = \frac{-eg_v g}{4\pi^2 m_s} = \frac{-eg_v}{4\pi^2 (2f_k - f_\pi)} \quad (29)$$

We used  $g_v = 5.5 \pm 0.5$  as estimated previously,  $f_\pi = 91.9 \pm 3.5 \text{ MeV}$ ,  $f_k = 112.9 \pm 1.27 \text{ MeV}$  [18] and  $\phi_p = (39.7 \pm 1.3)^\circ$  in the numerical calculations. The vector mixing angle  $\phi_v$  is extracted from the experimental value for  $\phi \rightarrow \pi^0\gamma$  which yields  $\phi_v = (3.31 \pm 0.42)^\circ$ . The theoretical uncertainties come mainly from these quantities. The experimental amplitudes are obtained from the corresponding branching ratios [18].

Decay	Amplitude( $A_{vp}$ )	$A_{vp}^{QL\sigma M} (GeV^{-1})$	$ A_{vp}^{EXP}  (GeV^{-1})$
$\rho^0 \rightarrow \pi^0\gamma$	$A$	$0.229 \pm 0.022$	$0.294 \pm 0.037$
$\rho^0 \rightarrow \eta\gamma$	$B \cos\phi_p$	$0.529 \pm 0.053$	$0.562 \pm 0.051$
$\omega \rightarrow \pi^0\gamma$	$B \cos\phi_v$	$0.687 \pm 0.067$	$0.711 \pm 0.016$
$\omega \rightarrow \eta\gamma$	$A \cos\phi_p \cos\phi_v + C \sin\phi_p \sin\phi_v$	$0.164 \pm 0.017$	$0.161 \pm 0.013$
$\eta' \rightarrow \rho^0\gamma$	$B \sin\phi_p$	$0.439 \pm 0.045$	$0.388 \pm 0.006$
$\eta' \rightarrow \omega\gamma$	$A \sin\phi_p \cos\phi_v - C \cos\phi_p \sin\phi_v$	$0.160 \pm 0.015$	$0.137 \pm 0.007$
$\phi \rightarrow \pi^0\gamma^*$	$B \sin\phi_v$	$0.039 \pm 0.001$	$0.039 \pm 0.001$
$\phi \rightarrow \eta\gamma$	$A \sin\phi_v \cos\phi_p - C \cos\phi_v \sin\phi_p$	$0.211 \pm 0.020$	$0.209 \pm 0.002$
$\phi \rightarrow \eta'\gamma$	$A \sin\phi_v \sin\phi_p + C \cos\phi_v \cos\phi_p$	$-0.233 \pm 0.023$	$0.225 \pm 0.025$

TABLE I. Amplitudes for the  $V \rightarrow P\gamma$  and  $P \rightarrow V\gamma$  decays. The process marked as (\*) is used as input to fix the vector mixing angle.

The  $V \rightarrow V'\gamma\gamma$  decays were firstly studied in [6] in the context of Heavy Vector Chiral Perturbation Theory (HVCHPT) [7]. There are two mechanisms contributing to these decays in this formalism. The first one is the decay chain  $V \rightarrow V'P \rightarrow V'\gamma\gamma$ . The second possibility is through loops of charged mesons. Furthermore, it was shown in [6] that these mechanisms do not interfere in the spin averaged decay rates.

On the experimental side, the decay  $\phi \rightarrow \rho\gamma\gamma$  has been already tested by the CMD-2 Coll. as a byproduct of their analysis of the  $\phi \rightarrow \pi^+\pi^-\pi^0$  decay [9]. Although they are not able to discriminate if the later decay proceeds through the chain  $\phi \rightarrow \rho\pi^0 \rightarrow \pi^+\pi^-\pi^0$  or it is a direct decay, they can distinguish events where the two final photons come from the  $\pi^0$  and pose an upper bound to the branching ratio for  $\phi \rightarrow \rho\gamma\gamma$  proceeding through other mechanisms. They obtain

$$BR(\phi \rightarrow \rho\gamma\gamma)_{exp} < 5 \times 10^{-4}. \quad (30)$$

In the context of HVCHPT, in addition to the  $\phi \rightarrow \rho\pi^0 \rightarrow \rho\gamma\gamma$  contributions which have been removed in Eq.[30], this decay can proceed only through loops of charged mesons. The corresponding branching ratios were calculated in [6] as

$$\begin{aligned} BR(\phi \rightarrow \rho\gamma\gamma)_{HVCHPT} &= 5.8 \times 10^{-9} \left(\frac{g_2}{0.75}\right)^4 \\ BR(\phi \rightarrow \omega\gamma\gamma)_{HVCHPT} &= 4.2 \times 10^{-9} \left(\frac{g_2}{0.75}\right)^4 \end{aligned} \quad (31)$$

where the low energy constant  $g_2$  is related to the  $VV'P$  couplings. Estimates for this constant using constituent quark model and chiral quark model yield  $g_2 \approx 1$  thus predicting extremely small branching ratios.

The  $\phi \rightarrow (\rho, \omega)\gamma\gamma$  decays were reconsidered in [8] in the framework of Vector Meson Dominance. In this framework, in addition to the possibilities appearing in HVCHPT, the  $V \rightarrow V'\gamma\gamma$  decay can proceed through the chain  $V \rightarrow P\gamma \rightarrow V'\gamma\gamma$ . The contributions from intermediate pseudoscalar states enhance the corresponding branching ratios to [8]

$$\begin{aligned} BR(\phi \rightarrow \rho\gamma\gamma)_{VMD} &\approx 1.3 \times 10^{-4} \\ BR(\phi \rightarrow \omega\gamma\gamma)_{VMD} &\approx 1.5 \times 10^{-5}, \end{aligned} \quad (32)$$

several orders of magnitude larger than those predicted by HVCHPT. The contributions of intermediate pseudoscalars do not appear in HVCHPT due to the conservation of heavy vector meson number inherent to this theory.

Let us consider these decays in the framework of  $QL\sigma M$ . In this formalism the  $V \rightarrow V'\gamma\gamma$  are induced by quark loops. As discussed in the previous section, the assumed low variation of the form factors from  $q^2 = 0 \rightarrow m_V^2$  is supported by the experimental data on  $V \rightarrow P^0\gamma$  and  $P^0 \rightarrow V\gamma$  decays. Here we will work with the same assumptions. Under these circumstances, our calculation is very similar to that performed in [8]. However, there are some differences which turn out to be relevant in the numerics. Firstly, our formalism incorporates  $SU(3)$  symmetry breaking, secondly we take into account mixing of vector mesons. The  $\phi \rightarrow (\omega, \rho)\gamma\gamma$  decays are dominated by the exchange of on-shell  $\eta'$ . Although there are also contributions coming from  $\pi$  and  $\eta$  exchange, the  $\phi \rightarrow \pi^0\gamma$  decay is suppressed by the OZI rule (see Table I) and  $\eta$  contributions turn out to be negligible as compared with the on-shell  $\eta'$  exchange and will not be considered here.

The invariant amplitude for  $\phi(q_1, \eta_1) \rightarrow \rho(q_2, \eta_2)\gamma(k_1, \varepsilon_1)\gamma(k_2, \varepsilon_2)$  is

$$\mathcal{M} = f_t \epsilon(\eta_1, \varepsilon_1, k_1, q_1) \epsilon(\eta_2, \varepsilon_2, k_2, q_2) + f_u \epsilon(\eta_1, \varepsilon_2, k_2, q_1) \epsilon(\eta_2, \varepsilon_1, k_1, q_2) \quad (33)$$

where

$$f_t = \frac{A_{\phi\eta'} A_{\rho\eta'}}{t - \tilde{m}_{\eta'}^2} \quad f_u = \frac{A_{\phi\eta'} A_{\rho\eta'}}{u - \tilde{m}_{\eta'}^2} \quad (34)$$

with

$$\tilde{m}_{\eta'}^2 = m_{\eta'}^2 - i \Gamma_{\eta'} m_{\eta'}, \quad s = (q_1 - q_2)^2, \quad t = (q_1 - k_1)^2, \quad u = (q_1 - k_2)^2. \quad (35)$$

This invariant amplitude corresponds to  $\eta'$  exchange in  $t$  and  $u$  channels. The average squared amplitude is

$$|\bar{\mathcal{M}}|^2 = \frac{1}{12} [ |f_t|^2 (t - M^2)^2 (t - m^2)^2 + |f_u|^2 (u - M^2)^2 (u - m^2)^2 + \text{Re}(f_t f_u^*) F(s, t, u) ] \quad (36)$$

where

$$F(s, t, u) = s^2 M^2 m^2 + (M^2 m^2 - tu)^2 \quad (37)$$

which is explicitly invariant under  $t \leftrightarrow u$  as required by Bose symmetry. Here,  $M, m$  stand for the masses of  $\phi$  and the final vector meson respectively. The decay width is given by

$$\frac{d\Gamma}{ds} = \frac{1}{(2\pi)^3} \frac{1}{32M^3} \frac{1}{2} \int_{t_0}^{t_1} |\bar{\mathcal{M}}|^2 dt \quad (38)$$

where

$$t_{0,1} = \frac{1}{2} [(M^2 + m^2 - s) \mp \sqrt{(M^2 + m^2 - s)^2 - 4m^2 M^2}]. \quad (39)$$

If we integrate numerically this equation using the experimental values for the  $\phi \rightarrow \eta' \gamma$ ,  $\eta' \rightarrow \rho \gamma$  and  $\eta' \rightarrow \omega \gamma$  couplings listed in Table I, and the values for the masses quoted in [18] we obtain the following branching ratios

$$BR(\phi \rightarrow \rho \gamma \gamma) = 2.3 \times 10^{-5}, \quad BR(\phi \rightarrow \omega \gamma \gamma) = 2.6 \times 10^{-6}. \quad (40)$$

If instead we use the values for the couplings as predicted by the model also listed in Table I we get

$$BR(\phi \rightarrow \rho \gamma \gamma)_{QL\sigma M} = 2.5 \times 10^{-5}, \quad BR(\phi \rightarrow \omega \gamma \gamma)_{QL\sigma M} = 2.8 \times 10^{-6}. \quad (41)$$

These results are roughly five times smaller than those reported in [8]. It is instructive to understand where the differences come from. To this end let us analyze the  $\phi \eta' \gamma$  coupling in both schemes. In our formalism, this coupling is given by

$$g_{\phi \eta' \gamma} = \frac{eg_v}{8\pi^2 f_\pi} \sin \phi_v \sin \phi_p - \frac{eg_v}{4\pi^2 (2f_k - f_\pi)} \cos \phi_v \cos \phi_p, \quad (42)$$

whereas the coupling used in [8], when we use  $\phi_p = \theta + \alpha$  where  $\sin \alpha = \sqrt{2/3}$ ,  $\cos \alpha = \sqrt{1/3}$ , can be written as

$$g_{\phi \eta' \gamma} = \frac{eg_v}{4\pi^2 f_\pi} \cos \phi_p. \quad (43)$$

Clearly, modulo an irrelevant global sign, our analytical results reduce to those presented in [8] when we take the SU(3) limit ( $f_k = f_\pi$ ) and consider ideal mixing for vector mesons  $\phi_v = 0$ . However, even if we take  $\phi_v = 0$ , the SU(3) corrections introduce a factor  $(\frac{2f_k - f_\pi}{f_\pi})^2 = (1.44)^2 = 2.04$  in the branching ratios. Further sources of discrepancy due to SU(3) symmetry breaking enter in the value used in the numerics for the pseudoscalar mixing angle. The extraction of this angle from  $\eta \rightarrow \gamma \gamma$  and  $\eta' \rightarrow \gamma \gamma$  is also sensitive to SU(3) symmetry breaking as can be seen in Eq.[23]. Our result,  $\phi_p = 39.7 \pm 1.3$ , is consistent with the recent measurement by the KLOE Coll.  $\phi_p^{exp} = 41.8_{-1.6}^{+1.9}$  [3] and is larger than the one used in [8], namely  $\phi_p = \theta + 54.7^\circ = 34.7^\circ$ . Additional sources of discrepancy come from the non-ideal vector mixing angle and the numerical value used for  $g_v$ . The importance of all these effects is clearly exhibited in the calculation of the branching ratio for the  $\phi \rightarrow \eta' \gamma$  decay. Indeed, this branching ratio was calculated to be  $2.1 \times 10^{-4}$  in [8]. In our formalism we obtain  $6.8 \times 10^{-5}$  to be compared with the recent measurement by the KLOE Coll. [3], namely  $BR(\phi \rightarrow \eta' \gamma)_{exp} = (6.1 \pm 0.61 \pm 0.43) \times 10^{-5}$ . In summary, SU(3) corrections are necessary for the correct description of  $\phi \eta' \gamma$ , hence they are also relevant for  $\phi \rightarrow (\rho, \omega) \gamma \gamma$  which are dominated by on-shell  $\eta'$  exchange. For the latter decays additional sources for the different numerics come from the  $\eta' \rho \gamma$  and  $\eta' \omega \gamma$  couplings, whose ratio deviates from the value of 3 predicted by SU(3) (see Table I), and from phase space integral.

In any case, the  $\phi \rightarrow \rho \gamma \gamma$  decay is still close to the upper bound in Eq.(30) and both decays are within the reach of DAΦNE.

## V. CONCLUSIONS

In this work we study the decays  $P^0 \rightarrow \gamma \gamma$ ,  $V^0 \rightarrow P^0 \gamma$  and  $V^0 \rightarrow V'^0 \gamma \gamma$ , in the framework of an  $U(3) \otimes U(3)$  linear sigma model which includes constituent quarks. All these decays are induced by quark loops in this model. For the

$P^0 \rightarrow \gamma\gamma$  decays we reproduce results previously obtained considering the contributions of external electromagnetic fields to the axial anomalies in the framework of the  $U(3) \otimes U(3)$  linear sigma model and using 't Hooft's anomaly matching conditions [15]. In the case of  $V^0 \rightarrow P^0\gamma$  decays there are spurious contributions coming from the opening of on-shell  $\bar{q}q$  pairs in the loops due to the lack of a mechanism for confinement. We avoid this problem assuming that the corresponding form factors are smoothly varying functions of  $q^2$  and calculating their value at  $q^2 = 0$ . Comparison with experimental results shown in Table I support this picture. The calculations for  $V^0 \rightarrow V'^0\gamma\gamma$  under these approximations improve those performed in [8] by incorporating effects due to  $SU(3)$  symmetry breaking and the mixing of vector mesons. Our numerical results turn out to be smaller than those obtained in [8] roughly by a factor of 5. This difference can be traced to  $SU(3)$  symmetry breaking, non-ideal mixing for vector mesons and numerical input -which manifests in their large prediction for the branching ratio of the  $\phi \rightarrow \eta'\gamma$  decay- and also to phase space numerical integration. Our predictions for branching ratios of the most interesting processes are:  $BR(\phi \rightarrow \rho\gamma\gamma)_{QL\sigma M} = 2.5 \times 10^{-5}$ ,  $BR(\phi \rightarrow \omega\gamma\gamma)_{QL\sigma M} = 2.8 \times 10^{-6}$ . In the former case the calculated  $BR$  is quite close to the upper bound obtained by the CMD-2 Coll.  $BR(\phi \rightarrow \rho\gamma\gamma)_{exp} < 5 \times 10^{-4}$  [9] and these decays could be detected for the first time at DAΦNE.

## VI. ACKNOWLEDGMENTS

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- [1] M. Takizawa and M. Oka, Phys. Lett. B **359**, 210 (1995) [Erratum-ibid. B **364**, 249 (1995)] [arXiv:hep-ph/9506254]; P. Ball, J. M. Frere and M. Tytgat, Phys. Lett. B **365**, 367 (1996) [arXiv:hep-ph/9508359]; A. Bramon, R. Escribano and M. D. Scadron, Phys. Lett. B **403**, 339 (1997) [arXiv:hep-ph/9703313]; A. Bramon, R. Escribano and M. D. Scadron, Eur. Phys. J. C **7**, 271 (1999) [arXiv:hep-ph/9711229]; A. Bramon, R. Escribano and M. D. Scadron, Phys. Lett. B **503**, 271 (2001) [arXiv:hep-ph/0012049]; H. M. Choi and C. R. Ji, Phys. Rev. D **59**, 074015 (1999) [arXiv:hep-ph/9711450]; L. Burakovsky and T. Goldman, Phys. Lett. B **427**, 361 (1998) [arXiv:hep-ph/9802404]; J. Cao, F. G. Cao, T. Huang and B. Q. Ma, Phys. Rev. D **58**, 113006 (1998) [arXiv:hep-ph/9807508]; L. Burakovsky and T. Goldman, arXiv:hep-ph/9808221; C. R. Ji and H. M. Choi, Acta Phys. Polon. B **29**, 3363 (1998); W. S. Carvalho, A. C. Antunes and A. S. de Castro, Eur. Phys. J. C **7**, 95 (1999); R. Escribano and J. M. Frere, Phys. Lett. B **459**, 288 (1999) [arXiv:hep-ph/9901405]; G. M. Shore, Nucl. Phys. B **569**, 107 (2000) [arXiv:hep-ph/9908217]; D. M. Li, H. Yu and Q. X. Shen, [arXiv:hep-ph/0010346]; E. Gedalin, A. Moalem and L. Razdolskaya, Phys. Rev. D **64**, 076007 (2001) [arXiv:hep-ph/0106301].
  - [2] R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Nucl. Phys. A **675**, 424C (2000); M. N. Achasov *et al.* [SND Collaboration], arXiv:hep-ex/9809013.
  - [3] A. Aloisio *et al.* [KLOE Collaboration], Phys. Lett. B **541**, 45 (2002) [arXiv:hep-ex/0206010];
  - [4] Phys. Lett. B **537**, 21 (2002) [arXiv:hep-ex/0204013]; Phys. Lett. B **536**, 209 (2002) [arXiv:hep-ex/0204012].
  - [5] J. Wess and B. Zumino, Phys. Lett. B **37**, 95 (1971); W. A. Bardeen, Phys. Rev. **184**, 1848 (1969).
  - [6] A. K. Leibovich, A. V. Manohar and M. B. Wise, Phys. Lett. B **358**, 347 (1995) [Erratum-ibid. B **376**, 332 (1996)] [arXiv:hep-ph/9508210].
  - [7] E. Jenkins, A. V. Manohar and M. B. Wise, Phys. Rev. Lett. **75**, 2272 (1995) [arXiv:hep-ph/9506356].
  - [8] P. Ko, J. Lee and H. S. Song, Phys. Lett. B **366**, 287 (1996) [arXiv:hep-ph/9510205].
  - [9] R. R. Akhmetshin *et al.*, Phys. Lett. B **434**, 426 (1998).
  - [10] J. L. Lucio-Martinez, M. Napsuciale, M. D. Scadron and V. M. Villanueva, Phys. Rev. D **61**, 034013 (2000); M. Napsuciale, J. L. Lucio Martinez and V. M. Villanueva, Frascati Physics Series Vol.15, 499 (1999), T. Bressani *et al.* Eds.
  - [11] S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. **41**, 531 (1969); M. Levy, Nuov. Cim. **LIIA**, 7247 (1967); J. Schechter and Y. Ueda, Pys. Rev. **D3**, 2874 (1971).
  - [12] M. Napsuciale and S. Rodriguez, Int. J. Mod. Phys. A **16**, 3011 (2001).
  - [13] N. A. Tornqvist, Eur. Phys. J. C **11**, 359 (1999) [arXiv:hep-ph/9905282].
  - [14] M. Napsuciale, A. Wirzba and M. Kirchbach, Nucl. Phys. A **703**, 306 (2002) [arXiv:nucl-th/0105055].
  - [15] M. Napsuciale and S. Rodriguez, Phys. Rev. D **65**, 096013 (2002) [arXiv:hep-ph/0112105].
  - [16] T. Feldmann, Int. J. Mod. Phys. A **15**, 159 (2000) [arXiv:hep-ph/9907491].
  - [17] R. Delbourgo, D. s. Liu and M. D. Scadron, Int. J. Mod. Phys. A **14**, 4331 (1999) [arXiv:hep-ph/9905501];
  - [18] K. Hagiwara *et al.* [Particle Data Group Collaboration], Phys. Rev. D **66**, 010001 (2002).